Sample Bayesian Probability Problem

Suppose that a drug test is 99% sensitive (i.e. it will correctly identify a drug user 99% of the time) and 99% specific (i.e. it will correctly identify a nonuser as testing negative 99% of the time). This seems like a pretty reliable test. Assume a test group, of which only 0.5% of members are drug users. Find the probability that, given a positive test, the subject is actually a drug user.

Let $+$ be a positive test and
$!+$ be a negative test

Let D stand for drug user and
$!D$ stand for non-user

What we know:

$P(D) = 0.005$ so $P(!D) = 0.995$

$P(+|D) = 0.99$
$P(!+|!D) = 0.99$

We want to calculate the probability that if someone has a $+$ test, s/he is actually a drug user: $P(D|+)$

This is related to $P(+|D)$ by Baye's rule: $P(D|+) = \dfrac{P(+|D)\,P(D)}{P(+)}$

0.99 0.005

$$P(D|+) = \frac{P(+|D)\,P(D)}{P(+)}$$

This is the tricky one.
The total probability of a positive test is the sum of the probabilities of getting a positive test AND being a user and of getting a positive test AND being a nonuser:

$$\begin{aligned} P(+) &= P(+ \cap D) + P(+ \cap !D) \\ &= P(+|D)\,P(D) + P(+|!D)\,P(!D) = 0.0149 \end{aligned}$$

0.99 0.005 0.995

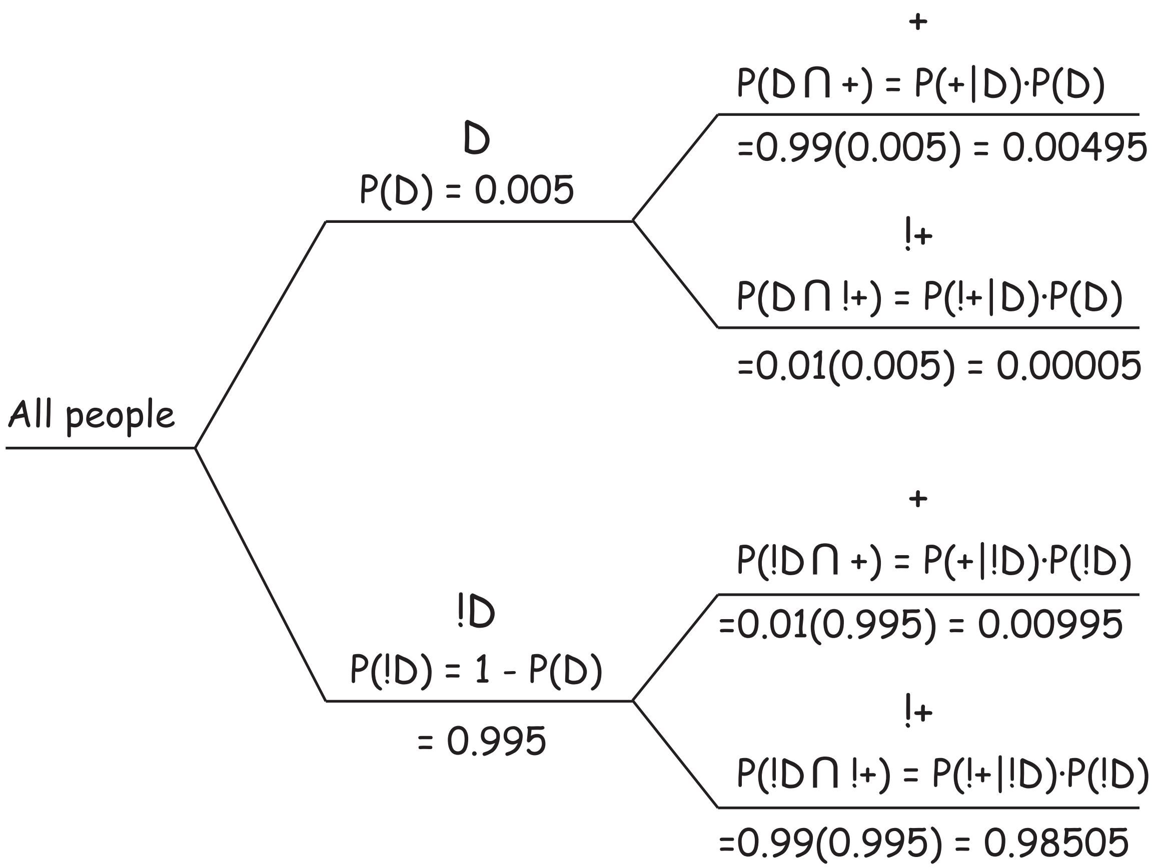
$= 1 - P(+|D) = 0.01$

because there are only two cases where we can have a + test, with or without a drug user.

so finally, $P(D|+) = \dfrac{(0.99)(0.005)}{(0.0149)} = 0.332$

$\boxed{\text{The chance that if someone has a positive test s/he is actually a drug user is only 33.2\%}}$

Sometimes it helps to visualize these problems with a tree diagram.



Total probability: 1.0 1.0